Holographic Double Diffractive Production of Higgs and the AdS Graviton/Pomeron

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The holographic approach to double diffractive Higgs production is presented for the AdS graviton/Pomeron of Brower, Polchinski, Strassler and Tan [1]. The goal is to provide a simple framework from the dual strong coupling point of view, which nonetheless is capable of providing phenomenologically compelling estimates of the cross sections. This article is the first step in defining the building block in anticipation of experimental observations at the LHC. As in the traditional weak coupling approach in order to constrain the phenomenological parameters, we anticipate the holographic parameterizations must subsequently be tested and calibrated through factorization for a self-consistent description of other diffractive process such as total cross sections, deep inelastic scattering and heavy quark production in the central region.

1 Introduction

A promising production mechanism for Higgs meson at the LHC involves the forward proton-proton scattering $pp \to pHp$. The protons scatter through very small angles with a large rapidity gaps separating the Higgs in the central region. The Higgs subsequently decays into large transverse momentum fragments. Although this represents a small fraction of the total cross section, the exclusive channel should provide an exceptional signal to background discrimination by constraining the Higgs mass to both the energy of decay fragments and the energy lost to the forward protons [2]. Relaxing the kinematics to allow for inclusive double diffraction may also be useful, where one or both of the nucleon are diffractively excited. While double diffraction is unlikely to be a discovery channel, it may play a useful role in determine properties of the Higgs after discovery.

Current phenomenological estimates of the diffractive Higgs production cross section have generally followed two approaches: perturbative (weak coupling) vs confining (strong coupling), or equivalently, in the Regge literature, often referred to as the "hard Pomeron" vs"soft Pomeron" methods. The Regge approach to high energy scattering, although well motivated phenomenologically, has suffered in the past by the lack of a precise theoretical underpinning. The advent of AdS/CFT has dramatically changed the

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situation. In a holographic approach, the Pomeron is a well-defined concept and it can be identified as the "AdS graviton" in the strong coupling [1], or, simply the BPST Pomeron. In this talk, we briefly review the general properties of the BPST Pomeron and then show how it can be used to describe double-diffractive production of Higgs.

2 Holographic Model for Diffractive Higgs Production

The formulation of AdS/CFT for high energy diffractive collision has already a rather extensive literature to draw on [3, 4, 5]. "Factorization in AdS" has emerged as a *universal* feature, applicable to scattering involving both particles and currents. For instance, for elastic scattering, the amplitude can be represented schematically in a factorizable form,

$$A(s,t) = \Phi_{13} * \widetilde{\mathcal{K}}_P * \Phi_{24}\ . \tag{1}$$

where Φ_{13} and Φ_{24} represented two elastic vertex couplings, and $\widetilde{\mathcal{K}}_P$ is an universal Pomeron kernel [1], with a characteristic power behavior at large $s >> |t|$,

$$\widetilde{\mathcal{K}}_P \sim s^{j_0}\ , \tag{2}$$

schematically represented by Fig. 1a. This "Pomeron intercept", j_0, lies in the range $1 < j_0 < 2$ and is a function of the 't Hooft coupling, $g^2 N_c$. The convolution in (1), denoted by the $*$-operation, involves an integration over the AdS location in the bulk. (For more details, see the talk by M. Djurić at this Workshop.) This formalism has also been applied to give a reasonable account of the small-x contribution to deep inelastic scattering [7]. In moving from elastic to DIS, one simply replaces Φ_{13} in (1) by appropriate product of propagators for external currents [6, 7].

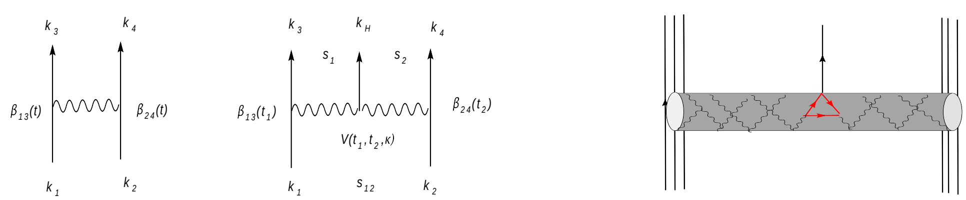

Figure 1: (a) Kinematics for single-Regge limit for 2-to-2 amplitudes, (b) Double-Regge kinematics for 2-to-3 amplitudes. (c) Cylinder Diagram for large N_c Higgs Production.

A holographic treatment of Higgs production amounts to a generalization of our previous AdS treatment for 2-to-2 amplitudes to one for 2-to-3 amplitudes, e.g., from Fig. 1a to Fig. 1b. A more refined analysis for Higgs production requires a careful treatment for that depicted in Fig. 1c. A particularly useful paper for the diffractive Higgs analysis is the prior work by Herzog, Paik, Strassler and Thompson [8]

[1] Unlike the case of a graviton exchange in AdS, this Pomeron kernel contains both real and imaginary parts.

on holographic double diffractive scattering. In this analysis, one generalizes (1) to 2-to-3 amplitude where

$$A(s, s_1, s_2, t_1, t_2) = \Phi_{13} * \widetilde{\mathcal{K}}_P * V_H * \widetilde{\mathcal{K}}_P * \Phi_{24} \ , \tag{3}$$

schematically represented by Fig. 1b. However, a new aspect, not addressed in [8], is the issue of scale invariance breaking. A proper accounting for a non-vanishing gluon condensate $\langle F^2 \rangle$ turns out to be a crucial ingredient in understanding the strength of diffractive Higgs production.

Let us first list the assumptions and the corresponding building blocks required to develop a model for holographic description of diffractive Higgs production. The basic theoretical steps necessary in order to arrive at (1) and (3) are:

(a) *Diffractive Scattering and QCD at Large N_c Limit*: in this limit, there is a more precise definition of the "bare Pomeron". In leading order of the $1/N_c$ expansion at fixed 't Hooft coupling $\lambda = g^2 N_c$, diffraction is given peturbatively by the exchange of a network of gluons with the topology of a cylinder, corresponding in a confining theory to the t-channel exchange of a closed string for glueball states. Such a state can be identified with the Pomeron.

(b) *From Weak to Strong Coupling*: Prior to AdS/CFT, property of the Pomeron has been explored mostly from a perturbative approach. The advent of the AdS/CFT correspondence has provided a firmer foundation from which a non-perturbative treatment can now be carried out. For instance, for elastic scattering, the 2-to-2 amplitude can be represented by the exchange of a single graviton, schematically given in a factorizable form,

$$A(s, t) = \Phi_{13} * \widetilde{\mathcal{K}}_G * \Phi_{24} \ . \tag{4}$$

where Φ_{13} and Φ_{24} represented two elastic vertex couplings to the graviton and $\widetilde{\mathcal{K}}_G$ is dominated by the "$(++,--)$" component of the graviton propagator [3]. Since this corresponds to a spin-2 exchange, the dominant graviton kernel $\widetilde{\mathcal{K}}_G$ grows with a integral power, i.e., at fixed t, as s^2. Similarly, double diffractive Higgs production will be dominated by a double-graviton exchange diagram, leading to a similar factorizable expression for the production amplitude

$$A(s, s_1, s_2, t_1, t_2) = \Phi_{13} * \widetilde{\mathcal{K}}_G * V_H * \widetilde{\mathcal{K}}_G * \Phi_{24} \ . \tag{5}$$

In comparing with (4), a new Higgs production vertex V_H is required. The central issue in a holographic description for diffractive Higgs production is the specification of this new vertex V_H.

(d) *Confinement*: However, above discussion is purely formal since a CFT has no scale and one needs to be more precise in defining the Regge limit. First, in order to provide a particle interpretation, the basic framework is a holographic approximation to the dual QCD with confinement deformation. With confinement deformation, the AdS is effectively cutoff. Because of the "cavity effect", both dilaton and the transverse-traceless metric become massive, leading to an infinite set of massive scalar and tensor glueballs respectively. In particular, each glueball state can be described by a normalizable wave function $\Phi(z)$ in AdS. The weight factor Φ_{ij} in the respective factorized representation for the elastic and Higgs amplitudes, (4) and (5), is given by $\Phi_{ij}(z) = e^{-2A(z)}\Phi_i(z)\Phi_j(z)$. In contrast, for amplitudes involving external currents, e.g., for DIS [6, 7], non-normalizable wave-functions will be used.

(e) *Correction to Strong Coupling in* $1/\sqrt{\lambda}$: It has been shown in [1], for $\mathcal{N}=4$ SUSY YM, the leading strong coupling Pomeron [1, 3, 4] is at

$$j_0 = 2 - 2/\sqrt{g^2 N_c}\,. \tag{6}$$

which is "lowered" from $J=2$ as one decreases λ. In a realistic holographic approach to high energy scattering, one must work at λ large but finite in order to account for the Pomeron intercept of the order $j_0 \simeq 1.3$. After taking into account $O(1/\sqrt{\lambda})$ correction to the Graviton kernel, one arrives at (1) and (3) for elastic and diffractive Higgs production respectively. Here the Pomeron kernel, $\widetilde{\mathcal{K}}_P$, has hard components due to near conformality in the UV and soft Regge behavior in the IR. It is interesting to compare the weak and strong coupling (conformal) Pomeron by plotting the intercept of the leading sigularity in the $J-$plane. This is to be compared with the weak coupling BFKL intercept to second order, as shown in Fig. 2. The phenomenological estimate for QCD gives an intercept of about $j_0 \simeq 1.3$, suggesting that the physics of diffractive scattering is roughly in the cross-over region between strong and weak coupling.

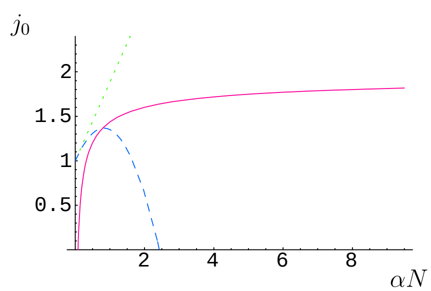

Figure 2: In $\mathcal{N}=4$ Yang-Mills theory, the weak- and strong-coupling calculations of the position j_0 of the leading singularity for $t \le 0$, as a function of $\alpha N = g^2 N_c/4\pi$. Shown are the leading-order BFKL calculation (dotted), the next-to-leading-order calculation (dashed), and the strong-coupling calculation of this paper (solid). Note the latter two can be reasonably interpolated.

(f) *Higgs Production From Weak to Strong Coupling*: In a perturbative approach, often dubbed as "hard Pomeron", Higgs production can be viewed as gluon fusion in the central rapidity region. A Higgs can be produced at central rapidity by the double Regge Higgs vertex through a heavy quark loop which in lowest order is a simple gluon fusion process, dominant for large parton x for the colliding gluons. A more elaborate picture emerges as one tries to go to the region of the softer (wee gluons) building up double Regge regime [2] In the large N_c there are no quark loop in the bulk of AdS space and since the Higgs in the Standard Model only couples to quark via the Yukawa interactions there appears to be a

[2]In addition to the Pomeron exchange contribution in these models must subsequently be reduced by large Sudakov correction at the Higgs vertex and by so called survival probability estimates for soft gluon emission, again reflecting the view that double diffraction Higgs production is intrinsically non-perturbative.

problem with strong coupling Higgs production in leading $1/Nc$. Fortunately the solution to this is to follow the standard procedure in Higgs phenomenology, which is to integrate out the quark field replacing the Higgs coupling to the gauge operator $Tr[F^2]$.

Consider the Higgs coupling to quarks via a Yukawa coupling, and, for simplicity we will assume is dominated by the top quark. We will be more explicit in the next Section, and simply note here that, after taking advantage of the scale separations between the QCD scale, i.e., the Higgs mass and the top quark mass, $\Lambda_{qcd} \ll m_H \ll 2m_t$, heavy quark decoupling allows one to replace the Yukawa coupling by an effective interaction,

$$\mathcal{L} = \frac{\alpha_s g}{24\pi M_W} F^a_{\mu\nu} F^{a\mu\nu} \phi_H \tag{7}$$

by evaluating the two gluon Higgs triangle graph in leading order $O(M_H/m_t)$. Now the AdS/CFT dictionary simply requires that this be the source in the UV of the AdS dilaton field. It follows, effectively, for Higgs production, we are required to work with a five-point amplitudes, one of the external leg involves a scalar dilaton current coupling to $Tr[F^2]$. For diffractive Higgs production, in the supergravity limit, the Higgs vertex V_H is given by a two-graviton-dilaton coupling, Fig. 1c.

(g) *Conformal Symmetry Breaking*: We now must pause to realize that in any conformal theory the is no dimensional parameter to allow for such a dimensionful two-graviton-dilaton coupling, $M^2\phi h_{\mu\nu}h^{\mu\nu}$, emerging in an expansion of the AdS gravity action if scale invariance is maintained. However since QCD is not a conformal theory this is just one of many reasons to introduce conformal symmetry breaking. Many attempts have been made to suplement this phenomenological Lagrangian with other fields such as the gauge fields for the light quark Goldstone modes to provide a better holographic dual for QCD. In principle enen at leading order of large N_c we should eventually require an infinite number of (higher spin) field in the bulk representation to correspond the yet undiscovered 2-d sigma model for the world-sheet string theory for QCD. Fortunately for the phenomenological level at high energy, these details are non-essential. To model an effective QCD background we will for the most part introduce two modifications of the pure AdS background: (1) an IR hardwall cut-off beyond $z = 1/\Lambda_{qcd}$ to give confinement and linear static quark potential at large distances and (2) a slow deformation in the UV ($z \to 0$) to model the logarithmic running for asymptotic freedom. Both break conformal invariance, which as we will argue is required to couple the two gravitons to the dilaton and produce a Higgs in the central rapidity region.

After taking into account of finite λ correction, the leading order Higgs production diagram at large N_c can be schematically represented in Fig. 1c, with each of the left- and right-cylinder representing a BPST Pomeron. It should be pointed out, just as in the case of elastic scattering, it is necessary to consider higher order corrections, e.g., eikonal corrections. We will not do it here, but will address this issue in the conclusion section. In what follows, we shall focus on the Pomeron-Pomeron fusion vertex in the strong coupling limit.

Finally it should be noted that one critical missing ingredient of these ad hoc conformal breaking deformation of the AdS geometry in the UV and IR is the fact the spontaneous breaking of pure Yang Mills (and presumable QCD at large N_c), via “dimensional transmutation” eliminates the coupling, λ, as a free parameter. It is fixed via the beta function in terms of a single integration constant (sometime called Λ_{qcd}) which provides the only mass scale. Thus the logarithmic scale violation in the UV are tied

to the same parameter giving confinement in the IR. All holographic modes of QCD to date introduce two mass scales and thus neglect this constraint. The solution to this problem also presumably awaits the determination of the unique string theory for large N_c QCD.

3 Pomeron-Pomeron fusion Vertex

We are now in a position to focus on the issue of double diffractive Higgs production from the perspective of String/Gauge duality, i.e., the Higgs vertex, V_H. It is important to stress that our general discussion in moving from single-Pomeron exchange processes, (1), to double-Pomeron exchange, (3), applies equally well for both diffractive glueball production and for Higgs production. The difference lies in how to treat the new central vertex. For the production of a glueball, the vertex will be proportional to a normalizable AdS wave-function. There will also be an overall factor controlling the strength of coupling to the external states, e.g., the Pomeron-Pomeron-glueball couplings. For Higgs production, on the other hand, the central vertex, V_H, involves a non-normalizable bulk-to-boundary propagator, appropriate for a scalar external current.This in turns leads to coupling to a Higgs scalar. The difference between these two cases parallels the situation for four-point amplitudes in moving from proton-proton (p-p) elastic scattering to electron-proton deep-inelastic scattering (e-p DIS). In moving from p-p to DIS, one simply replaces one of the two pairs of normalizable proton wave-functions with a pair of non-normalizable counterparts appropriate for conserved external vector currents.

A Higgs scalar in the standard model couples exclusively to the quarks via Yukawa coupling, which for simplicity we will assume is dominated by the top quark, with

$$\mathcal{L} = -\frac{g}{2M_W} m_t\ \bar{t}(x)t(x)\phi_H(x). \tag{8}$$

Taking advantage of the scale separations between the QCD scale, the Higgs mass and the top quark mass, $\Lambda_{qcd} \ll m_H \ll 2m_t$, heavy quark decoupling allows one to replace the Yukawa coupling by direct coupling of Higgs to gluons, which is treated as an external source in the AdS dictionary. Consequently V_H, in a coordinate representation, is replaced by the vertex for two AdS Pomerons fusing at $(x'_{1\perp}, z'_1)$ and $(x'_{2\perp}, z'_2)$ and propagating this disturbance to the $\bar{t}(x)t(x)$ scalar current at the boundary of AdS. The double diffractive Higgs vertex V_H can then be obtained in a two-step process.

First, since the Yukawa Higgs quark coupling is proportional to the quark mass, it is dominated by the top quark. Assuming $m_H \ll m_t$, this can be replaced by an effective interaction, (7), by evaluating the two gluon Higgs triangle graph in leading order $O(M_H/m_t)$. Second, using the AdS/CFT dictionary, the external source for $F^a_{\mu\nu}F^a_{\mu\nu}(x)$ is placed at the AdS boundary ($z_0 \to 0$) connecting to the Pomeron fusion vertex in the interior of AdS_3 at $\mathbf{b}_H = (x'_H, z'_H)$, by a scalar bulk-to-boundary propagator, $K(x'_H - x_H, z'_H, z_0)$.

We are finally in the position to put all the pieces together. Although we eventually want to go to a coordinate representation in order to perform eikonal unitarization, certain simplification can be achieved more easily in working with the momentum representation. The Higgs production amplitude,

schematically given by (3), can then be written explicitly as

$$\begin{aligned} A(s,s_1,s_2,t_1,t_2) &\simeq \int dz_1 dz dz_2 \sqrt{-g_1}\sqrt{-g}\sqrt{-g_2}\,\Phi_{13}(z_1) \\ &\times \widetilde{\mathcal{K}}_P(s_1,t_1,z_1,z)\,V_H(q^2,z)\,\widetilde{\mathcal{K}}_P(s_2,t_2,z,z_2)\,\Phi_{24}(z_2)\ . \end{aligned} \tag{9}$$

where $q^2 = -m_H^2$. For this production vertex, we will keep it simple by expressing it as

$$V_H(q^2,z) = V_{PP\phi} K(q^2,z) L_H\ . \tag{10}$$

where $K(q^2,z)$ is the conventionally normalized bulk to boundary propagator, $V_{PP\phi}$ serves as an overall coupling from two-Pomeron to F^2, and L is the conversion factor from F^2 to Higgs, i.e., $L_H = L(-m_H^2) \simeq \frac{\alpha_s g}{24\pi M_W}$. By treating the central vertex $V_{PP\phi}$ as a constant, which follows from the super-gravity limit, we have ignored possible additional dependence on κ, as well as that on t_1 and t_2. This approximation gives an explicit factorizable form for Higgs production.

4 Strategy for Phenomenological Estimates

While we intend to lay in this article the formal framework for the holographic diffractive Higgs production approach, it is useful to outline the phenomenological approach we plan to pursue to confront experimental data. There should be a strong warning however that details will necessarily change as we discover which parameterization are critical to a global analysis of data. Our current version for the holographic Higgs amplitude involves 3 parameters: (1) the IR cut-off determined by the glueball mass, (2) the leading singularity in the J-plane determined [3] by the 't Hooft parameter $g^2 N_c$ and (3) the strength of the central vertex parameterized by the string coupling or Planck mass. A strategy must be provided in fixing these parameters.

As a first step in this direction, we ask how the central vertex, V_H, or equivalently, $V_{PP\phi}$, via (10), can be normalized, following the approach of Kharzeev and Levin [2] based on the analysis of trace anomaly. We also show how one can in principle use the elastic scattering to normalize the bare BPST Pomeron coupling to external protons and the 't Hooft coupling $g^2 N_c$. As in the case of elastic scattering, it is pedagogically reasonable to begin by first treating the simplest case of double-Pomeron exchange for Higgs production, i.e., without absorptive correction. We discuss how phenomenolgically reasonable simplifications can be made. This is followed by treating eikonal corrections in the next section, which provides a means of estimating the all-important survival probability.

4.1 Continuation to Tensor Glueball Pole and On-Shell Higgs Coupling:

Confinement deformation in AdS will lead to glueball states, e.g., the lowest tensor glueball state lying on the leading Pomeron trajectory [9]. There will also be scalar glueballs associated with the dilaton.

[3] In a true dual to QCD, there is no independent parameter for the strong coupling, because of "dimensional transmutation", which fixes all dimensionful quantities relative to the a single mass scale Λ_{qcd}, through the running coupling constant. For instance, the glueball mass in units of Λ_{qcd} is fixed and computed in lattice computations.

With scalar invariance broken, this will also lead to non-vanishing couplings between a pair of tensor glueballs and scalar glueballs. In terms of the language of Witten diagram, corresponds to a non-vanishing graviton-graviton-dilaton coupling in the bulk, which in turn leads to $V_H \neq 0$.

Consider first the elastic amplitude. With confinement, each Pomeron kernel will contain a tensor glueball pole when t goes on-shell. Indeed, the propagator for our Pomeron kernel can be expressed as a discrete sum over pole contributions. That is, when $t \simeq m_0^2$, where m_0 is the mass of the lightest tensor glueball, which lies on the leading Pomeron trajectory. In this limit, the elastic amplitude then takes on the expected pole-dominated form,

$$A(s,t) \simeq g_{13}\, \frac{s^2}{m_0^2 - t}\, g_{24} \tag{11}$$

with vertex g_{ij} given by an overlapping integral: $g_{ij}(m_0^2) = \alpha' \int dz \sqrt{-g(z)} e^{-4A(z)}\ \beta(m_0^2)\Phi_i(z)\Phi_j(z)\ \phi_G(z)$. Here $\phi_G(z)$ is the wave function for the tensor glueball. We have also generalized Φ_{ij} by writing it as $\Phi_{ij}(t,z) = \beta(t) e^{-2A(z)}\Phi_i(z)\Phi_j(z)$ for phenomenological reasons. That is, the external coupling g_{ij} is given by an overlap-integral over a product of three wave functions, $\Phi_i(z)$, $\Phi_j(z)$ and $\phi_G(z)$. With the standard normalization, $A(s,t)$ is dimensionless.

A similar analysis can also be carried out for the Higgs production amplitude, Eq. (9). Note that the Pomeron kernel now appears twice, $\widetilde{\mathcal{K}}_P(s_1,t_1,z_1,z)$ and $\widetilde{\mathcal{K}}_P(s_2,t_2,z_2,z)$. When nearing the respective tensor poles at $t_1 \simeq m_0^2$ and $t_2 \simeq m_0^2$, the amplitude can be expressed as

$$A(s,s_1,s_2,t_1,t_2) \quad \simeq \quad g_{13}\, \frac{\Gamma_{GGH}\ s^2}{(t_1 - m_0^2)(t_2 - m_0^2)}\, g_{24} \tag{12}$$

As for the elastic case, we have performed the z_1 and z_2 integrations, and have also made use of the fact that $s_1 s_2 \simeq \kappa\ s \simeq m_H^2 s$. Here Γ_{GGH} is the effective on-shell glueball-glueball-Higgs coupling, which can also be expressed as

$$\Gamma_{GGH} = L_H F(-m_H^2) \tag{13}$$

where $L_H = \frac{\alpha_s g}{24\pi M_W}$ and F is a scalar form factor $F(q^2) = \langle G, ++, q_1 | F^a_{\mu\nu} F^a_{\mu\nu}(0) | G, --, q_2\rangle$. That is, in the high energy Regge limit, the dominant contribution comes from the maximum helicity glueball state [1], with $\lambda = 2$. In this limit, this form factor, is given by the overlap of the dilaton bulk to boundary propagator

$$F(q^2) \quad = \quad (\alpha' m_H^2)^2 V_{PP\phi} \int dz \sqrt{-g(z)} e^{-4A(z)} \phi_G(z) K(q,z) \phi_G(z) \tag{14}$$

What remains to be specified is the overall normalization, $F(0)$.

We next follow D. Kharzeev and E. M. Levin [2], who noted that, from the SYM side, $F(q^2)$ at $q^2 = 0$, can be considered as the glueball condensate. Consider matrix elements of the trace-anomaly between two states, $|\alpha(p)\rangle$ and $|\alpha'(p')\rangle$, with four-momentum transfer $q = p - p'$. In particular, for a single particle state of a tensor glueball $|G(p)\rangle$, this leads to $\langle G(p)|\Theta^\alpha_\alpha|G(p')\rangle = \frac{\widetilde{\beta}}{2g}\langle G(p)|F^a_{\mu\nu}F^{a\mu\nu}|G(p')\rangle$. At $q = 0$, the forward matrix element of the trace of the energy-momentum tensor is given simply by the mass of the relevant tensor glueball, with $\langle G|\Theta^\alpha_\alpha|G\rangle = M_G^2$, this directly yields

$$F(0) = \langle G|F^a_{\mu\nu}F^{a\mu\nu}|G\rangle = -\frac{4\pi M_G^2}{3\widetilde{\beta}} \tag{15}$$

where $\widetilde{\beta} = -b\alpha_s/(2\pi)$, $b = 11 - 2n_f/3$, for $N_c = 3$. In what follows, we will use $n_f = 3$. Note that heavy quark contribution is not included in this limit. Since the conformal scale breaking is due the running coupling constant in QCD, there is apparently a mapping between QCD scale breaking and breaking of the AdS background in the IR, which gives a finite mass to the glueball and to give a non-zero contribution to the gauge condensate.

4.2 Extrapolation to the Near-Forward limit:

To apply the above result to the physical region, one needs to extrapolate from t near the tensor pole to the physical region where $t \simeq 0$. Let us first treat the elastic amplitude. A key difference, as one moves from $t \simeq m_0^2$ to $t \simeq 0$, is the fact that the amplitude becomes complex, with the leading s-dependence slowing down from s^2 to s^{j_0}, $1 < j_0 < 2$. To carry out this analysis, it is necessary to result to the J-plane representation for the Pomeron kernel $\tilde{\mathcal{K}}_P(s,t,z,z')$, with the J-plane propagator $\widetilde{G}_j(t,z,z')$ given by a sum of J-plane poles, e.g., for hardwall model. For our current purpose, it is sufficient to keep the contribution coming from the effective leading trajectory,

$$\widetilde{G}_j(z,z';t) \simeq \widetilde{\phi}_{eff}(z,j)\frac{1}{m_{eff}^2(j)-t}\widetilde{\phi}_{eff}(z',j) \ . \tag{16}$$

where we approximate the BPST-cut contribution by that of an effective leading pole, with the Pomeron kernel behaving as $s^{j_{eff}(t)}$, where $j_{eff}(t)$, the trajectory function, determined by $m_{eff}^2(j(t)) = t$. That is, we assume $j_{eff}(t)$ remains real in the physical region where $t < 0$. By performing the inverse Mellin transform, the large s-behavior of the BPST kernel can easily be obtained, leading to

$$A(s,t) \simeq g_{13}(t)\ \left(\frac{\xi(j_{eff}(t))\ (\alpha' s)^{j_{eff}(t)}}{\alpha'^2 \widetilde{m}^2(t)}\right)\ g_{24}(t) \tag{17}$$

where $\widetilde{m}^2$ is the inverse of trajectory slope, $\widetilde{m}^2(t) \equiv dm_{eff}^2(j(t))/dj$, and $\xi(j)$ is the signature factor. For the elastic amplitude, the coupling

$$g_{ij}(t) = \alpha' \int dz \sqrt{-g(z)}\ \Phi_{ij}(t,z) e^{-j_{eff}(t)A(z)}\widetilde{\phi}_0(z, j_{eff}(t)) \tag{18}$$

is again in the form of an overlapping integral over the product of three wave functions, with $\widetilde{\phi}_{eff} = \widetilde{\phi}_0(z, j_{eff}(t))$. This serves as a continuation away from the on-shell spin-2 exchange by replacing the spin-2 wave function $\phi_G(z) = \widetilde{\phi}_0(z,2)$ by a corresponding wave function for a Pomeron, $\widetilde{\phi}_0(z, j_{eff}(t))$, with spin shifted from 2 to $j_{eff}(t)$. Although this shift is of the order $O(1/\sqrt{\lambda})$, it is important to note that $\widetilde{\phi}_0(z, j(t)) \sim z^{\Delta(j(t))-2}$, for $z \to 0$, in contrast to $\widetilde{\phi}_0(z,2) \sim z^2$. Note that we have continued with the convention where $g_{ij}(t)$ has the dimension of length.

As one further continues to the physical region where $t \leq 0$, the amplitude will now be dominated by the contribution from the BPST cut, with the inverse Mellin transform in J turning into an integral over the discontinuity across the cut, $(-\infty, j_0)$. Since the contribution from a cut is no longer factorizable, it leads to diffusion in the AdS-radius. Analytic expression is available in the conformal limit, and, with

a hardwall, a similar analysis can also be carried out by parametrizing the leading Regge singularity, e.g., $m(j) \simeq m_0 + (m_0 - m_1)\left(\sqrt{\sqrt{\lambda}(j-j_0)/2} - 1\right)$, leading to our AdS representation for the elastic amplitude in the forward limit, Eq. (1).

4.3 First Estimate for Double-Pomeron Contribution to Differential Cross Section

For our present purpose, it is adequate to first ignore diffusion by adopt a simpler ansatz for the elastic amplitude by freezing the AdS radius at $z_0 \sim 1/\Lambda_{QCD}$. That is, we assume, for protons, wave-functions are concentrated near z_0, we can replace the Pomeron kernel with z and z' evaluated at z_0, i.e., $\tilde{\mathcal{K}}_P(t,s,z,z') \to \tilde{\mathcal{K}}_P(t,s,z_0,z_0)$ with the resulting z and z' integration leading to unity.

Focusing next on the forward limit $t = 0$, we denote the effective intercept by $\bar{j}_0$ and inverse slope by $\widetilde{m}^2$. Together with the forward coupling $g_{ij}(0)$, they will be determined phenomenologically. We note that $\widetilde{m}^2$ can be chosen to be of the order of the tensor glueball mass, m_0^2. For consistency, we also assume that $\bar{j}_0 \simeq j_0$. A corresponding treatment at $t_1 \simeq t_2 \simeq 0$ for the Higgs production amplitude, Eq. (9), can lead to a similar simplification. It follows, after a bit of algebra,

$$A(s,s_1,s_2,t_1 \simeq 0, t_2 \simeq 0) \quad \simeq \quad g_{13}(0)\frac{\xi(\bar{j}_0)^2\Gamma_{PPH}\,(\alpha' s)^{\bar{j}_0}}{(\alpha'\widetilde{m}^2)^2}\; g_{24}(0) \tag{19}$$

with an effective central vertex, related to $V_{PP\phi}$ by

$$\Gamma_{PPH} \quad \simeq \quad \frac{\alpha_s g}{24\pi M_W} V_{PP\phi}\left(\alpha' m_H^2\right)^{\bar{j}_0} C(\bar{j}_0) \tag{20}$$

where

$$C(\bar{j}_0) = \int dz\sqrt{-g}e^{-4A(z)}\widetilde{\phi}_0(z,\bar{j}_0)K(-m_H^2,z)\widetilde{\phi}_0(z,\bar{j}_0) \tag{21}$$

and we have dropped terms lower order in $O(1/\sqrt{\lambda})$. We point out that (21) is finite due to the wave-function normalizability. For hard-wall, it is logarithmically divergent as $\bar{j}_0 \to j_0$ which corresponds to the onset of a Regge cut. In a proper treatment when the leading singularity is a cut, this apparent divergence will be absent. In order to avoid complicating the discussion, we proceed with the understanding that $C(\bar{j}_0)$ is of the order unity.

Let us turn next to the non-forward limit. We accept the fact that, in the physical region where $t < 0$ and small, the cross sections typically have an exponential form, with a logarithmic slope which is mildly energy-dependent. We therefore approximate all amplitudes in the near forward region where $t < 0$ and small, $A(s,t) \simeq e^{B_{eff}(s)\,t/2}\,A(s,0)$ where $B_{eff}(s)$ is a smoothly slowly increasing function of s, (we expect it to be logarithmic). We also assume, for $t_1 < 0$, $t_2 < 0$ and small, the Higgs production amplitude is also strongly damped so that

$$A(s,s_1,s_2,t_1,t_2) \simeq e^{B'_eff(s_1)\,t_1/2}e^{B'_eff(s_2)\,t_2/2}\,A(s,s_1,s_2,t_1 \simeq 0, t_2 \simeq 0) \tag{22}$$

We also assume $B'_{eff}(s) \simeq B_{eff}(s) + \mathrm{b}$. With these, both the elastic, the total pp cross sections and the Higgs production cross section can now be evaluated. Various cross sections will of course depend

on the unknown slope parameter, B_{eff}, which can at best be estimated based on prior experience with diffractive estimates.

The phase space for diffractive Higgs production can be specified by the rapidity of Higgs y_H, and two-dimensional transverse momenta $q_{i,\perp}$, $i = 3, 4, 5$, with $q_{5,\perp} = q_{H,\perp}$, in a frame where the incoming momenta k_1 and k_2 are longitudinal. Alternatively, due to momentum conservation, we can use instead $y_H, t_1, t_2, \cos\phi$ as four independent variables where $t_1 \simeq -q_{3,\perp}^2$, $t_2 \simeq -q_{4,\perp}^2$, and $\cos\phi = \hat{q}_{3,\perp} \cdot \hat{q}_{4,\perp}$. However, the amplitude is effectively independent of ϕ since its dependence enters through the κ variable where $\kappa \simeq m_H^2 + q_{H,\perp}^2 = m_H^2 + (q_{2,\perp} + q_{4,\perp})^2$. As discussed earlier, for Higgs production, we can replace κ by $\kappa_{eff} \simeq m_H^2$.

Following the earlier analysis, it is now possible to provide a first estimate for the double-diffractive Higgs production. It is possible to adopt an approach advocated in by Kharzeev and Levin where the dependence on B_{eff} can be re-expressed in terms of other physical observables. Under our approximation, it is easy to show that the ratio $\sigma_{el}/\sigma_{total}^2$ can be expressed as $\frac{\sigma_{el}}{\sigma_{total}^2} = \frac{1+\rho^2}{16\pi B_{eff}(s)}$ where $\rho \equiv \mathrm{Re}\,\mathcal{K}(0, s, z_0, z_0)/\mathrm{Im}\,\mathcal{K}(0, s, z_0, z_0)$. Equivalently, one can relate B_{eff} directly in terms of the experimentally smooth dimensionless ratio, $R_{el}(s) = \sigma_{el}/\sigma_{total} = \frac{(1+\rho^2)\sigma_{total}(s)}{16\pi B_{eff}(s)}$. Upon squaring the amplitude, $A(s, s_1, s_2, t_1, t_2)$, (22), the double-differential cross section for Higgs production can now be obtained. After integrating over t_1 and t_2 and using the fact that, for m_H^2 large $s \simeq s_1 s_2/m_H^2$, one finds

$$\frac{d\sigma}{dy_H} \simeq (1/\pi) \times C' \times |\Gamma_{GGH}(0)/\widetilde{m}^2|^2 \times \frac{\sigma(s)}{\sigma(m_H^2)} \times R_{el}^2(m_H\sqrt{s}) \tag{23}$$

In this expression above, both C' and $\widetilde{m}^2$, like m_0^2, are model dependent. It is nevertheless interesting to note that, since $\Gamma_{GGH}(0) \sim m_0^2$, the glueball mass scale also drops out, leaving a model-dependent ratio of order unity. In deriving the result above, we have replaced B'_{eff} by B_{eff} where the difference is unimportant at high energy. With m_H in the range of $100GeV$, R_{el} can be taken to be in the range 0.1 to 0.2. For $C' \simeq 1$, we find $\frac{d\sigma}{dy_H} \simeq .8 \sim 1.2$ pbarn. This is of the same order as estimated in [2]. However, as also pointed in [2], this should be considered as an over-estimate. The major source of suppression will come from absorptive correction, which can lead to a central production cross section in the femtobarn range. We turn to this next.

5 Discussion

We conclude by discussing how consideration of higher order contributions via an eikonal treatment leads to corrections for the central Higgs production. Following by now established usage, the resulting production cross section can be expressed in terms of a "survival probability".

Although the "bare Pomeron" approximation dominates in the large N_c expansion, it is clear that higher order summations are necessary in order to restore unitarity. In flat space Veneziano has shown that higher closed string loops for graviton scattering eikonalize. Indeed in Refs. [3, 4] it was shown that the same sum leads to an eikonal expansion that exponentiates for each string bit frozen in impact parameter during the collision. To be more explicit, the resulting eikonal sum leads to an impact representation for

the 2-to-2 amplitude

$$A(s, x^\perp - x'^\perp) \quad = \quad -2is \int dz\ dz'\ P_{13}(z) P_{24}(z') \left[e^{i\chi(s,x^\perp - x'^\perp, z, z')} - 1 \right] \tag{24}$$

The eikonal χ, as a function of $x_\perp - x'_\perp$, z, z' and s, can be determined by matching the first order term in χ to the single-Pomeron contribution. In impact space representation, and one finds $\chi(s, x_\perp - x'_\perp, z, z') = \frac{g_0^2}{2s}\, \widetilde{\mathcal{K}}(s, x_\perp - x'_\perp, z, z')$

This eikonal analysis can be extended directly to Higgs production. To simplify the discussion, we shall adopt a slightly formal treatment. Since Higgs is not part of the QCD dynamics, one can formally treat our eikonal as a functional of a weakly coupled external background Higgs field, $\phi_H(q^\pm, x_H^\perp, z_H)$, that is, in (24), we replace $A(s, x_\perp, x'_\perp)$ and $\chi(s, x^\perp - x'^\perp, z, z')$ by $A(s, x^\perp, x'^\perp; \phi_H)$ and $\chi(s, x^\perp - x'^\perp, z, z'; \phi_H)$, with the understanding that they reduce to $A(s, x_\perp, x'_\perp)$ and $\chi(s, x^\perp - x'^\perp, z, z')$ respectively in the limit $\phi_H \to 0$. Since Higgs production is a small effect, by expanding to first order in the Higgs background field, we find the leading order Higgs production amplitude, to all order in χ, becomes

$$\begin{aligned} A_H(s_1, s_2, x^\perp - x_H^\perp, x'^\perp - x_H^\perp, z_H) \quad &= \quad 2s \int dz\ dz'\ P_{13}(z) P_{24}(z') \\ &\times \quad \chi_H(s_1, s_2, x^\perp - x_H^\perp, x'^\perp - x_H^\perp, z, z', z_H)\ e^{i\chi(s,x^\perp - x'^\perp, z, z')} \end{aligned} \tag{25}$$

where χ_H can be found by matching in the limit $\phi_H \to 0$ with the Higgs production amplitude, (9), due to double-Pomeron exchange in an impact representation. The net effect of eikonal sum is to introduce a phase factor $e^{i\chi(s,x_\perp - x'_\perp, z, z')}$ into the production amplitude. Due to its absorptive part, Im $\chi > 0$, this eikonal factor provides a strong suppression for central Higgs production.

The effect of this suppression is often expressed in terms of a "Survival Probability", $\langle S \rangle$. In a momentum representation, the cross section for Higgs production per unit of rapidity in the central region is $\frac{d\sigma_H(s,y_H)}{dy_H} = \frac{1}{\pi^3 (16\pi)^2 s^2} \int d^2 q_{1\perp} d^2 q_{2\perp} |A_H(s, y_H, q_{1\perp}, q_{2\perp})|^2$ where y_H is the rapidity of the produced Higgs, $q_{1\perp}$ and $q_{2\perp}$ are transverse momenta of two outgoing fast leading particle in the frame where the momenta of incoming particles are longitudinal. "Survival Probability" is conventionally defined by the ratio

$$\langle S \rangle \equiv \frac{\int d^2 q_{1\perp} d^2 q_{2\perp} |A_H(s, y_H, q_{1\perp}, q_{2\perp})|^2}{\int d^2 q_{1\perp} d^2 q_{2\perp} |A_H^{(0)}(s, y_H, q_{1\perp}, q_{2\perp})|^2} \tag{26}$$

where $A_H^{(0)}$ is the corresponding amplitude before eikonal suppression, e.g., given by Eq. (9). For simplicity, we shall also focus on the mid-rapidity production, i.e., $y_H \simeq 0$ in the overall CM frame. In this case, $\langle S \rangle$ is a function of overall CM energy squared, s, or the equivalent total rapidity, $Y \simeq \log s$. Evaluating the survival probability as given by (26), though straight forward, is often tedious. The structure for both the numerator and the denominator is the same. For numerator factor, one has

$$\begin{aligned} &\int dx_\perp dz\ d\bar{z}\ P_{13}(z) P_{13}(\bar{z}) \int dx'_\perp dz'\ d\bar{z}'\ P_{24}(z) P_{24}(z') \int e^{i\left(\chi(s,x_\perp - x'_\perp, z, z') - \chi^*(s, x_\perp - x'_\perp, \bar{z}, \bar{z}')\right)} \\ &\chi_H(s, s_1, s_2, x^\perp - x_H^\perp, x'^\perp - x_H^\perp, z, z') \chi_H^*(s, s_1, s_2, x^\perp - x_H^\perp, x'^\perp - x_H^\perp, \bar{z}, \bar{z}') \end{aligned} \tag{27}$$

where we have made use of that fact that $z_H \simeq 1/m_H$. To obtain the denominator, one simply removes the phase factor, $e^{i\left(\chi(s,x_\perp,x'_\perp,z,z')-\chi^*(s,x_\perp,x'_\perp,\bar{z},\bar{z}')\right)}$. It is now clear that it is this extra factor which controls the strength of suppression.

To gain a qualitative estimate, let us consider the local limit where $z \simeq \bar{z} \simeq z_0$ and $z' \simeq \bar{z}' \simeq z'_0$, with $z_0 \simeq z'_0 \simeq 1/\Lambda_{QCD}$. In this limit, one finds that this suppression factor reduces to

$$e^{-2\ \mathrm{Im}\ \chi(s,x_\perp,x'_\perp,z_0,z'_0)} \tag{28}$$

where Im $\chi > 0$ by unitarity. If follows that, in a super-gravity limit of strong coupling where the eikonal is strictly real, there will be no suppression and the survival probability is 1. Conversely, the fact that phenomenologically a small survival probability is required is another evidence of we need to work in an intermediate region where $1 < j_0 < 2$. In this more realistic limit, Im χ is large and cannot be neglected. In particular, it follows that the dominant region for diffractive Higgs production in pp scattering comes from the region where

$$\mathrm{Im}\ \ \chi(s, x_\perp - x'_\perp, z, z') = O(1), \tag{29}$$

with $z \simeq z' = O(1/\Lambda_{qcd})$. Note that this is precisely the edge of the "disk region" for p-p scattering. In order to carry out a quantitative analysis, it is imperative that we learn the property of $\chi(s, \vec{b}, z)$ for $|\vec{b}|$ large. From our experience with pp scattering, DIS at HERA, etc., we know that confinement will play a crucial role. In pp scattering, since $z \simeq z' = O(1/\Lambda_{qcd})$, we expect this condition is reached at relatively low energy, as is the case for total cross section. It therefore plays a dominant role in determining the magnitude of diffractive Higgs production at LHC. We will not discuss this issue here further; more pertinent discussions on how to determine $\chi(s, x_\perp - x'_\perp, z, z')$ when confinement is important can be found in Ref. [7].

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